

Mathematics: analysis and approaches

Higher level

Paper 1

8 May 2023

Zone A afternoon | Zone B morning | Zone C afternoon

Candidate session number

2 hours

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

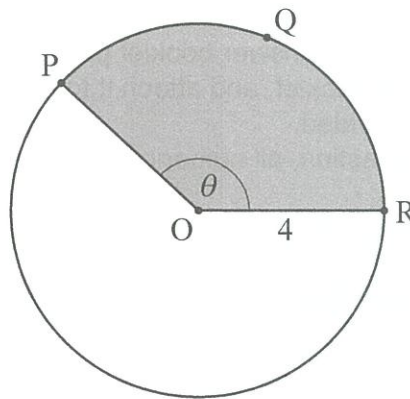
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 4 cm.

diagram not to scale



The points P , Q and R lie on the circumference of the circle and $\widehat{POR} = \theta$, where θ is measured in radians.

The length of arc PQR is 10 cm.

(a) Find the perimeter of the shaded sector. [2]

(b) Find θ . [2]

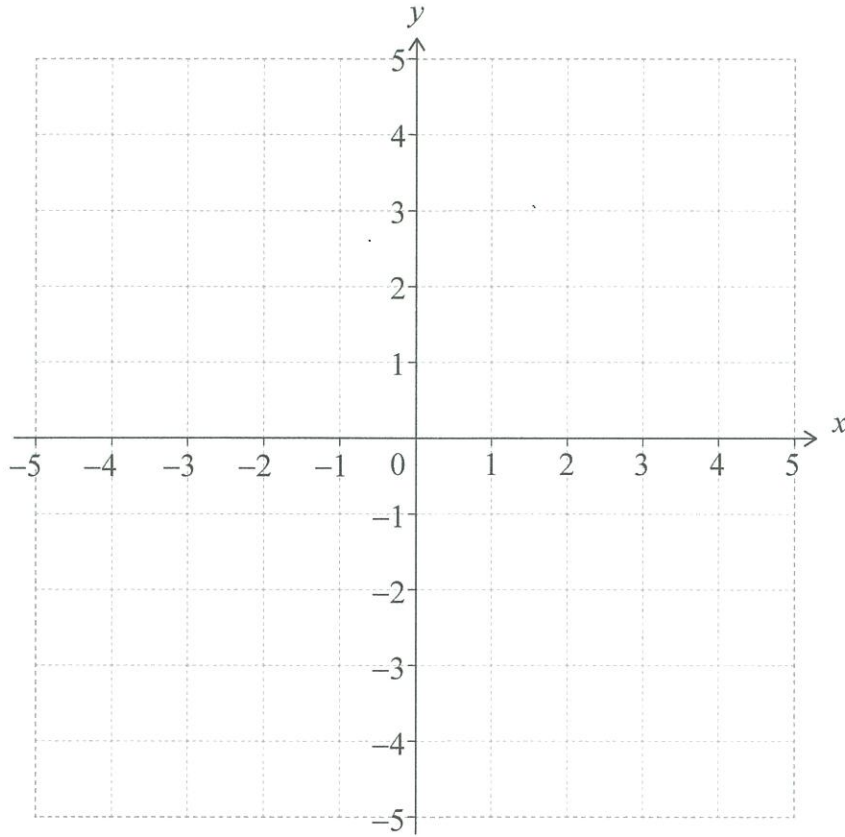
(c) Find the area of the shaded sector. [2]

(This question continues on the following page)

(Question 2 continued)

- (c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).

[1]



Turn over

Do **not** write solutions on this page.

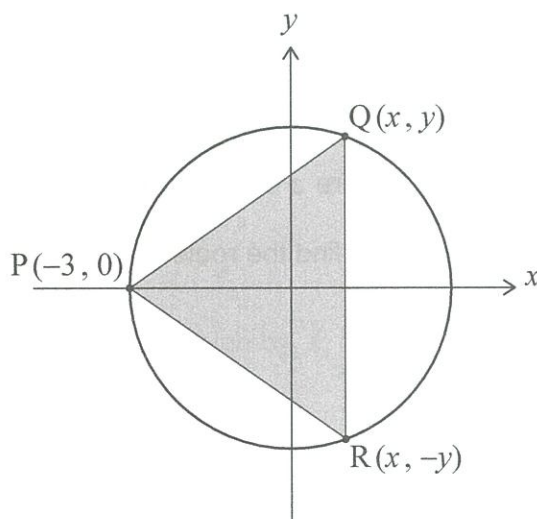
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 14]

A circle with equation $x^2 + y^2 = 9$ has centre $(0, 0)$ and radius 3.

A triangle, PQR, is inscribed in the circle with its vertices at $P(-3, 0)$, $Q(x, y)$ and $R(x, -y)$, where Q and R are variable points in the first and fourth quadrants respectively. This is shown in the following diagram.



- (a) For point Q, show that $y = \sqrt{9 - x^2}$. [1]
- (b) Hence, find an expression for A , the area of triangle PQR, in terms of x . [3]
- (c) Show that $\frac{dA}{dx} = \frac{9 - 3x - 2x^2}{\sqrt{9 - x^2}}$. [4]
- (d) Hence or otherwise, find the y -coordinate of R such that A is a maximum. [6]

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11. [Maximum mark: 22]

Consider the complex number $u = -1 + \sqrt{3}i$.

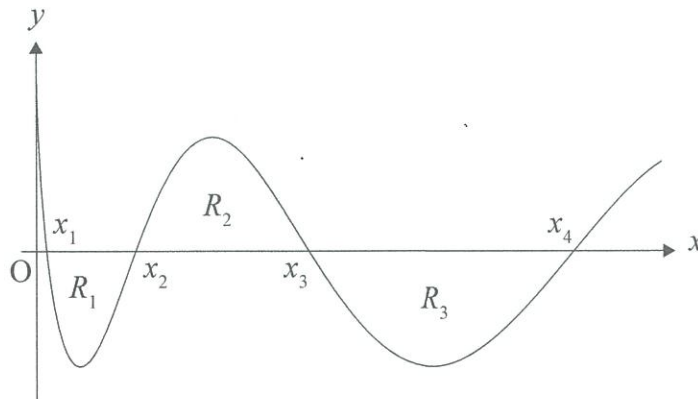
- (a) By finding the modulus and argument of u , show that $u = 2e^{i\frac{2\pi}{3}}$. [3]
- (b) (i) Find the smallest positive integer n such that u^n is a real number.
(ii) Find the value of u^n when n takes the value found in part (b)(i). [5]
- (c) Consider the equation $z^3 + 5z^2 + 10z + 12 = 0$, where $z \in \mathbb{C}$.
(i) Given that u is a root of $z^3 + 5z^2 + 10z + 12 = 0$, find the other roots.
(ii) By using a suitable transformation from z to w , or otherwise, find the roots of the equation $1 + 5w + 10w^2 + 12w^3 = 0$, where $w \in \mathbb{C}$. [9]
- (d) Consider the equation $z^2 = 2z^*$, where $z \in \mathbb{C}$, $z \neq 0$.
By expressing z in the form $a + bi$, find the roots of the equation. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 17]

(a) By using an appropriate substitution, show that $\int \cos \sqrt{x} \, dx = 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$. [6]

The following diagram shows part of the curve $y = \cos \sqrt{x}$ for $x \geq 0$.



The curve intersects the x -axis at $x_1, x_2, x_3, x_4, \dots$

The n th x -intercept of the curve, x_n , is given by $x_n = \frac{(2n-1)^2 \pi^2}{4}$, where $n \in \mathbb{Z}^+$.

(b) Write down a similar expression for x_{n+1} . [1]

The regions bounded by the curve and the x -axis are denoted by R_1, R_2, R_3, \dots , as shown on the above diagram.

(c) Calculate the area of region R_n .

Give your answer in the form $kn\pi$, where $k \in \mathbb{Z}^+$. [7]

(d) Hence, show that the areas of the regions bounded by the curve and the x -axis, R_1, R_2, R_3, \dots , form an arithmetic sequence. [3]